EE447-11

Large Scale Path Loss
Log Normal Shadowing

1

The Flat Fading Channel

- The channel functions are random processes and hard to characterize
- We therefore use the channel correlation functions
- Now assume:
  - The channel impulse response is a random variable
  - We describe the channel at any time t using a pdf

Consider a flat fading channel – where the delay spread is small compared with the symbol duration.

2

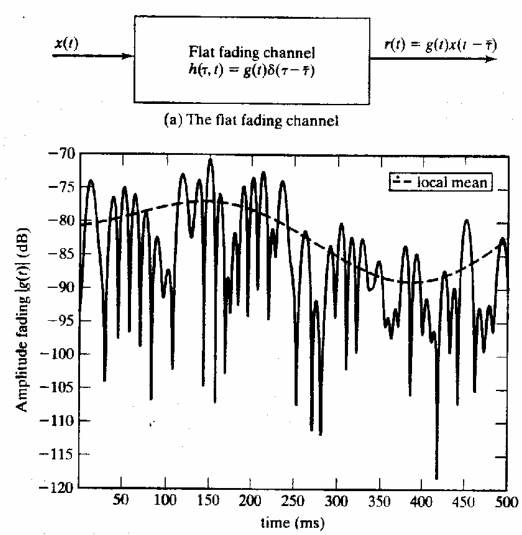
The Flat Fading Channel

FLAT FADING CHANNEL

- The delay spread does not effect the received signal
- The channel delay function is reduced to the mean delay $\bar{\tau}$ $\delta(t-\bar{\tau})$
- The channel exhibits a time-varying gain g(t)
- g(t) has a "short term fading" component Z(t) due to multipath. It is modeled statistically by a Rayleigh, Rician, or Nakagami disitribution and is independent of the distance between the transmitter and the receiver.
- g(t) also has a "long-term" path loss component that is the mean of g(t)

3

Large scale path loss



(b) Overall amplitude fading |g(t)| (dB)

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The Flat Fading Channel

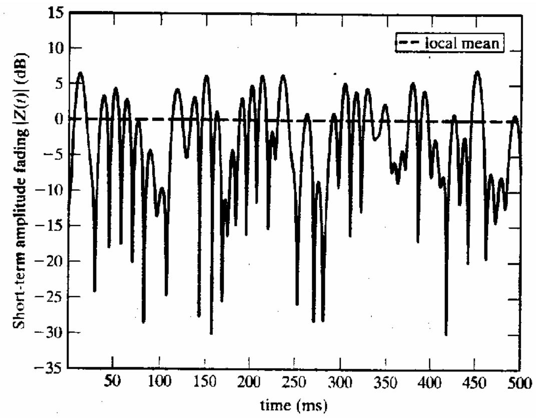

Figure 2.15 Representation of long-term and short-term fading components.

The dashed line is the mean (m) path loss.
The variation about the mean is described by the Normal Distribution

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Plane Earth Loss*-(again)

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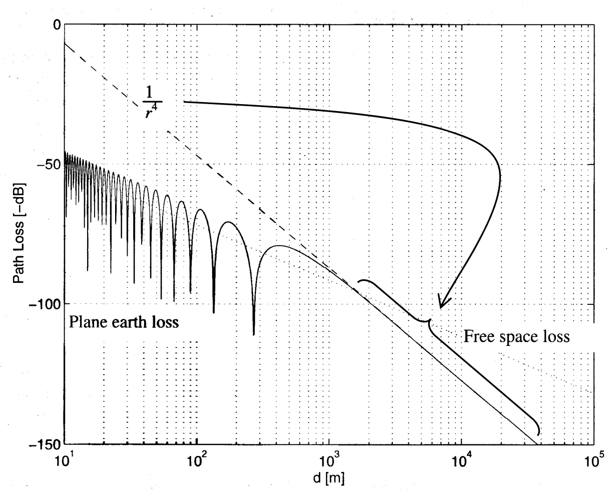

"Antennas and Propagation for Wireless Communication Systems" by Saunders

Figure 5.6: Plane earth loss (–), free space loss (…), approximate plane earth loss (– –) from (5.34). Here $h_m = 1.5$ m, $h_b = 30$ m, $f = 900$ MHz

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Log-Distance PL with Shadowing – mean path loss

$$\overline{L_p}(d) \propto \left(\frac{d}{d_0}\right)^k, \ for\ d \ge d_0$$

$$\overline{L_p} = \overline{L_p}(d_0) + 10kLog_{10}\left(\frac{d}{d_0}\right) dB,\ for\ d \ge d_0$$

$d_0 \cong 1\text{km}$ for macrocells, 1m indoors

Typical Path Loss Exponents for Different Environments	
Environment	Path loss Exponent, κ
free space	2
urban cellular radio	2.7 to 3.5
shadowed urban cellular radio	3 to 5
in building with LOS	1.6 to 1.8
obstructed in building	4 to 6

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Log-Distance PL with Shadowing – Statistical component to loss

The total path loss $L_p(d)$ with shadowing is then:

$$L_p(d) = \bar{L}_p(d) + \epsilon_{(dB)}$$
$$= \bar{L}_p(d_0) + 10\kappa \log_{10}\left(\frac{d}{d_0}\right) + \epsilon_{(dB)} \text{ (dB)}, \quad d \ge d_0.$$

Statistical path loss component

$$f_\epsilon(y) = \frac{20/\ln 10}{\sqrt{2\pi}\, y\sigma_\epsilon} \exp\left[-\frac{(20\log_{10} y)^2}{2\sigma_\epsilon^2}\right]. \quad (2.4.16)$$

The first-order statistics of log-normal shadowing are characterized by the standard deviation σ_ϵ, which can be obtained from measurements. For example, 8 dB is a typical value for σ_ϵ in an outdoor cellular system and 5 dB is a value for an indoor environment.

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Log-Distance PL with Shadowing

The total path loss $L_p(d)$ with shadowing is then:

$$L_p(d) = \overline{L_p} + \varepsilon_{dB} \; dB$$

ε_{dB} is the statistical variation of the path loss

$$\overline{L_p} = \overline{L_p}(d_0) + 10k Log_{10}\left(\frac{d}{d_0}\right)$$ is the mean path loss

$$L_p(d) \le L_{p\max}$$ for the TR link to perform correctly

note that for $\overline{L_p}$, d_0 and *k* may be adjusted to model Friss loss, plane earth loss, or any of the other models for mean path loss at a given frequency.

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Log-Distance PL with Shadowing

Shadowing: When the line of site path is blocked by an obstruction such as a building or a hill that is much larger than the λ of the signal

Long term fading is then a combination of the <u>log-distance path loss</u> and the **<u>log-normal shadowing</u> -that is statistical**. Let ε(dB) be a zero-mean Gaussian distributed random variable (d dB) with a standard deviation σ_ε (in dB). The pdf of ε(dB) is given as:

$$f_\varepsilon(x) = \frac{20/\ln(10)}{\sqrt{2\pi}\sigma_\varepsilon} e^{\left[-\frac{x^2}{2\sigma_\varepsilon{}^2}\right]}$$

A variable transform of x will give ε in a linear scale and is is sad to follow a log-normal distribution with pdf:

$$f_\varepsilon(y) = \frac{20/\ln(10)}{\sqrt{2\pi}\, y\sigma_\varepsilon} e^{\left[-\frac{(20\log_{10} y)^2}{2\sigma_\varepsilon{}^2}\right]}$$

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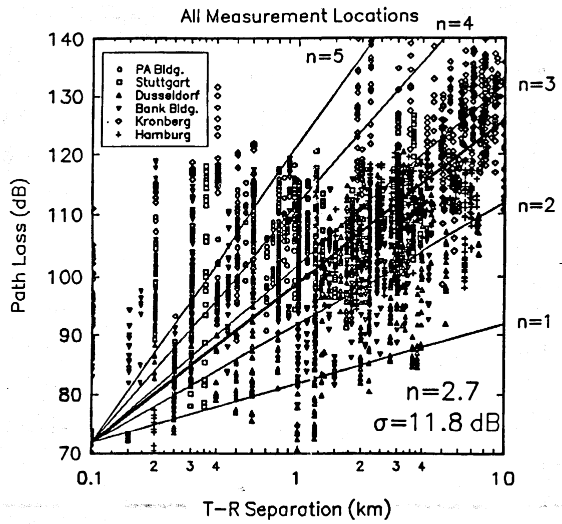
Log Normal Shadowing*



*Rappaport

Figure 4.17 Scatter plot of measured data and corresponding MMSE path loss model for many cities in Germany. For this data, $n = 2.7$ and $\sigma = 11.8$ dB [from [Sei91] © IEEE].

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Log-Distance PL with Shadowing

- The statistical term is a log-normal distribution
- This distribution is fully defined by the mean, m=0, and the standard deviation, σ_ε in dB
- What we want to determine is the $P(\varepsilon_{dB}) > \varepsilon_{dBmax}$

$$f_\varepsilon(y) = \frac{20/\ln(10)}{\sqrt{2\pi}\, y\sigma_\varepsilon} e^{\left[-\frac{(20\log_{10} y)^2}{2\sigma_\varepsilon^{\,2}}\right]}$$

- σ_ε is approximately 8 to10 dB outdoors, and 4 to 6 dB in a typical room.
- refer to the class discussion for the calculation of σ_ε from field data.

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EE447

Propagation –

Small Scale Multipath Fading

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Review: The Normal, Gaussian, Distribution

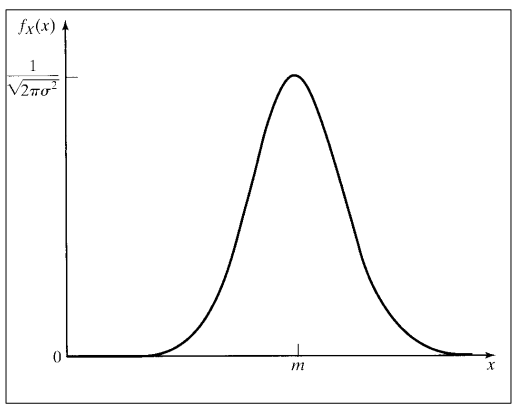
PDF: $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}$

CDF: $\Phi_X(x) = P(X \le x)$

$= \int_{-\infty}^{x} f_X(t)dt$

normalized, $m = 0, \;\; \sigma = 1$:

$\Phi_X(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}} dt$



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Review: The Normal Distribution

$$P(X > x) = 1 - \Phi_X(x) = Q(x)$$

$$Q(-x) = 1 - (Q(x)$$

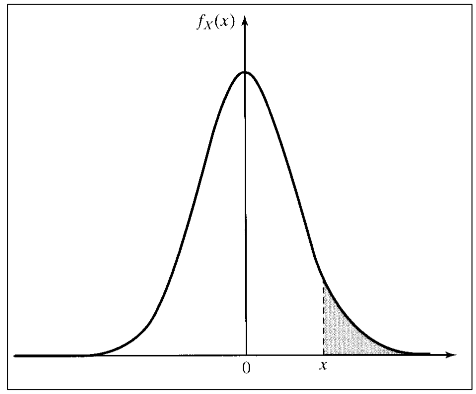
$$Q(0) = \tfrac{1}{2}$$

$$Q(\infty) = 0$$

Upper Bounds:

$$Q(x) \le \frac{1}{2} e^{\frac{-x^2}{2}} \quad x \ge 0$$

$$or \;\; Q(x) < \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \quad x \ge 0$$



The Q-function as the area under the tail of the Normal pdf

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Review: The Normal or Gaussian Distribution

$$Q(x) = P(X > x)$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{\frac{-x^2}{2}} dx$$

complementary error function:

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-x^2} dx = 2Q(x\sqrt{2})$$

$$Q(x) = \frac{1}{2} erfc(\frac{x}{\sqrt{2}}) = \frac{1}{2}\left[1 - erf(\frac{x}{\sqrt{2}}\right]$$

where erf is the error function (check your calulator)

$$P(X > x) = Q\left(\frac{x-m}{\sigma}\right)$$

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The Log Normal Distribution in path loss

Given a Normal Distribution with mean=m and standard deviation= σ :

$$P(X > x) = Q\left(\frac{x - m}{\sigma}\right)$$

- *m* is the mean path loss as shown in previous slides
- σ_ε is approximately 8 to10 dB outdoors, and 4 to 6 dB in a typical room.
- refer to the class discussion for the calculation of σ_ε from field data.

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Review - The Q function

TABLE 5.1 TABLE OF THE Q FUNCTION

0	5.000000e–01	2.4	8.197534e–03	4.8	7.933274e–07
0.1	4.601722e–01	2.5	6.209665e–03	4.9	4.791830e–07
0.2	4.207403e–01	2.6	4.661189e–03	5.0	2.866516e–07
0.3	3.820886e–01	2.7	3.466973e–03	5.1	1.698268e–07
0.4	3.445783e–01	2.8	2.555131e–03	5.2	9.964437e–06
0.5	3.085375e–01	2.9	1.865812e–03	5.3	5.790128e–08
0.6	2.742531e–01	3.0	1.349898e–03	5.4	3.332043e–08
0.7	2.419637e–01	3.1	9.676035e–04	5.5	1.898956e–08
0.8	2.118554e–01	3.2	6.871378e–04	5.6	1.071760e–08
0.9	1.840601e–01	3.3	4.834242e–04	5.7	5.990378e–09
1.0	1.586553e–01	3.4	3.369291e–04	5.8	3.315742e–09
1.1	1.356661e–01	3.5	2.326291e–04	5.9	1.817507e–09
1.2	1.150697e–01	3.6	1.591086e–04	6.0	9.865876e–10
1.3	9.680049e–02	3.7	1.077997e–04	6.1	5.303426e–10
1.4	8.075666e–02	3.8	7.234806e–05	6.2	2.823161e–10
1.5	6.680720e–02	3.9	4.809633e–05	6.3	1.488226e–10
1.6	5.479929e–02	4.0	3.167124e–05	6.4	7.768843e–11
1.7	4.456546e–02	4.1	2.065752e–05	6.5	4.016001e–11
1.8	3.593032e–02	4.2	1.334576e–05	6.6	2.055790e–11
1.9	2.871656e–02	4.3	8.539898e–06	6.7	1.042099e–11
2.0	2.275013e–02	4.4	5.412542e–06	6.8	5.230951e–12
2.1	1.786442e–02	4.5	3.397673e–06	6.9	2.600125e–12
2.2	1.390345e–02	4.6	2.112456e–06	7.0	1.279813e–12
2.3	1.072411e–02	4.7	1.300809e–06		

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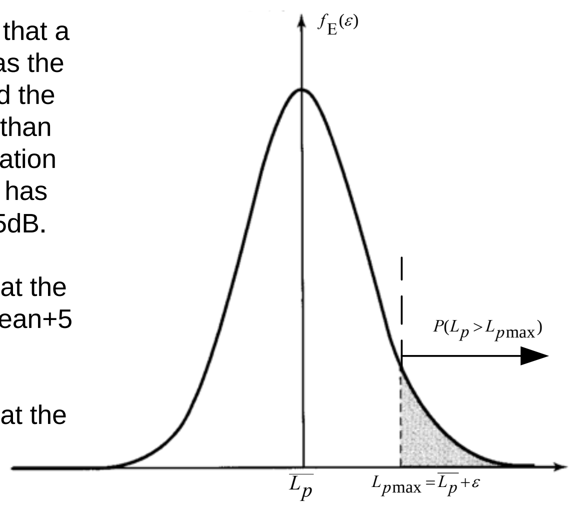
Log-Normal PL with Shadowing-Example

Example:
It has been determined that a link will operate as long as the path lossdoes not exceed the mean path loss by more than 5 dB. The standard deviation of the path loss variation has been determined to σ_ε=5dB.

What is the probability that the path loss will exceed Lmean+5 dB?

What is the probability that the path loss will exceed Lmean+10 dB?



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Log-Distance PL with Shadowing-Example

Example:
It has been determined that a link will operate as long as the path loss does not exceed the mean path loss by more than 5 dB. The standard deviation of the path loss variation has been determined to σ_ε=5dB. What is the probability that the path loss will exceed Lp+5 dB? What is the probability that the path loss will exceed Lp+10 dB?

$P(\varepsilon > 5dB)$ given $\sigma_\varepsilon = 5dB$ is

$$P(\varepsilon > 5) = Q\left(\frac{\varepsilon}{\sigma_\varepsilon}\right) = Q\left(\frac{\varepsilon = 5}{\sigma_\varepsilon = 5}\right)$$

$P(\varepsilon > 5) = Q(1)$

from the Q table

$Q(1) = 0.159 = \underline{15.9\%}$

$P(\varepsilon > 10dB)$ given $\sigma_\varepsilon = 5dB$ is

$$P(\varepsilon > 10) = Q\left(\frac{\varepsilon}{\sigma_\varepsilon}\right) = Q\left(\frac{\varepsilon = 10}{\sigma_\varepsilon = 5}\right)$$

$P(\varepsilon > 10) = Q(2)$

from the Q table

$Q(2) = 0.0228 = \underline{2.3\%}$

This is why links are often designed so that the mean received power is about 10 dB above the minimum power required for proper operation

Multipath Propagation

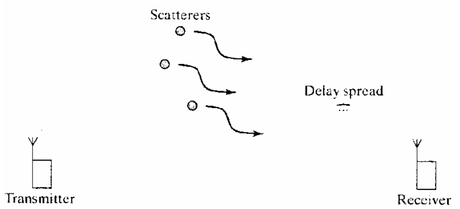

Figure 2.1 Multipath spread due to channel scattering.

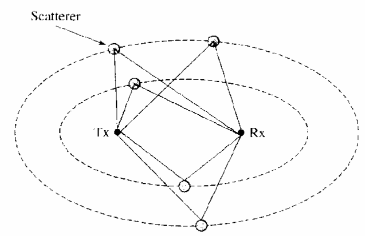

Figure 2.2 Ellipsoidal portrayal of scatterer location.

Multipath Propagation

Time Dispersion. Because multiple propagation paths have different propagation delays, the transmitted point source will be received as a smeared waveform. Nonoverlapping scatterers give rise to distinct multiple paths, which are characterized by their locations in the scattering medium. As depicted in Figure 2.2, all scatterers are located on ellipses with the transmitter (Tx) and receiver (Rx) as the foci. One ellipse is associated with one path length. Therefore, signals reflected by scatterers located on the same ellipse will experience the same propagation delay. The signal components from these multiple paths are indistinguishable at the receiver. Signals that are reflected by scatterers located on different ellipses will arrive at the receiver with differential delays. If the maximum differential delay spread is small compared with the symbol duration of the transmitted signal, the channel is said to exhibit flat fading. If the differential delay spread is large compared with the symbol interval, the channel exhibits frequency-selective fading. In the time domain, the received signals corresponding to successive transmitted symbols will overlap, giving rise to a phenomenon known as intersymbol interference (ISI). ISI is a signal-dependent distortion. The severity of ISI increases with the width of the delay spread. The ISI distortion in the time domain can also be examined in the frequency domain. ISI degrades transmission performance. Channel equalization techniques can be used to combat ISI, as discussed in Chapter 4.

Multipath Propagation

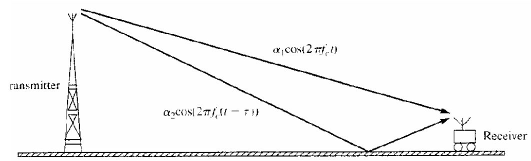

Figure 2.3 A channel with two propagation paths.

Amplitude and Phase of the Received Signal Vary

$$r(t) = \alpha_1 \cos(2\pi f_c t) + \alpha_2 \cos(2\pi f_c (t - \tau)),$$

$$r(t) = \alpha \cos(2\pi f_c t + \phi),$$

where

$$\alpha = \sqrt{\alpha_1^2 + \alpha_2^2 + 2\alpha_1\alpha_2 \cos(2\pi f_c \tau)}$$

and

$$\phi = -\tan^{-1}\left[\frac{\alpha_2 \sin(2\pi f_c \tau)}{\alpha_1 - \alpha_2 \cos(2\pi f_c \tau)}\right]$$

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Multipath Propagation

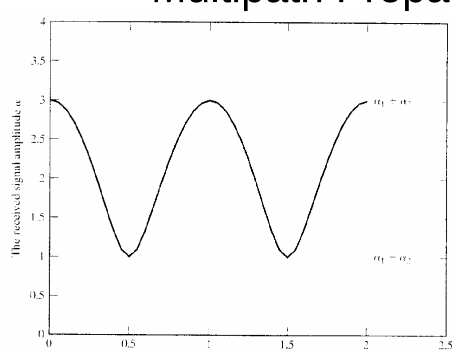

Figure 2.4 The amplitude fluctuation of the two-path channel with $\alpha_1 = 2$ and $\alpha_2 = 1$.

In summary, multipath propagation in the wireless mobile environment results in a fading dispersive channel. The signal propagation environment changes as the mobile station moves and/or as any surrounding scatterers move. Therefore, the channel is time varying and can be modeled as a linear time-variant (LTV) system.

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Time-Variant Transfer Function Impulse Response

Definition 2.1 The impulse response of an LTV channel, $h(\tau, t)$, is the channel output at t in response to an impulse applied to the channel at $t - \tau$.

In Definition 2.1, the variable τ represents the propagation delay. From the definition and Eq. (2.2.5), the channel output can be represented in terms of the impulse response and the channel input by

$$r(t) = \int_{-\infty}^{\infty} h(\tau, t)x(t - \tau)\,d\tau. \tag{2.2.6}$$

The channel impulse response for the channel with N distinct scatterers is then

$$h(\tau, t) = \sum_{n=1}^{N} \alpha_n(t)e^{-j\theta_n(t)}\delta(\tau - \tau_n(t)). \tag{2.2.7}$$

Phase may change more rapidly than Amplitude

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Time-Variant Transfer Function Impulse Response

2.2.2 Time-Variant Transfer Function

With the multipath channel characterized as a linear system, the channel behavior can also be examined in the frequency domain via a Fourier transformation. Time and frequency have an inverse relationship.

Definition 2.2 The time-variant transfer function of an LTV channel is the Fourier transform of the impulse response, $h(\tau, t)$, with respect to the delay variable τ.

Let $H(f, t)$ denote the channel transfer function, as shown in Figure 2.6. We have the Fourier transform pair

$$\begin{cases} H(f, t) = \mathcal{F}_\tau[h(\tau, t)] = \int_{-\infty}^{\infty} h(\tau, t)e^{-j2\pi f\tau}d\tau \\ h(\tau, t) = \mathcal{F}_f^{-1}[H(f, t)] = \int_{-\infty}^{\infty} H(f, t)e^{+j2\pi f\tau}df \end{cases}.$$

where the time variable t can be viewed as a parameter. The received signal can be represented in terms of the transmitted signal and the transfer function as

$$r(t) = \int_{-\infty}^{\infty} R(f, t)e^{j2\pi ft}df, \tag{2.2.8}$$

where

$$R(f, t) = H(f, t)X(f)$$

and

$$X(f) = \mathcal{F}[x(t)].$$

At any instant, say $t = t_0$, the transfer function $H(f, t_0)$ characterizes the channel in the frequency domain. As the channel changes with t, the frequency domain representation also

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Time-Variant Transfer Function Impulse Response

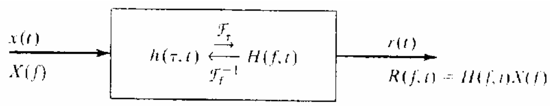

Figure 2.6 Frequency-time channel representation.

changes with t. Therefore, we have the channel time-varying transfer function. If the channel is time invariant, then the impulse response is a function of the delay variable τ and is independent of the time variable t; thus the transfer function varies only with the frequency variable f and is independent of t. This is consistent with the impulse response and transfer function of an LTI channel.

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Small-Scale Multipath Fading

be the amplitude fading and carrier distortion introduced by the channel. The fading characteristics can be studied by examining the pdfs of the envelope $\alpha(t)$ and phase $\theta(t)$ at any time t. The fading characteristics depend on whether the transmitter and receiver are in line-of-sight or not in line-of-sight. The former case is called LOS scattering while the latter case is referred to as NLOS scattering. LOS scattering has a specular component (from the direct path), and can be modeled as a Rician distribution. NLOS scattering does not have a specular component, and can be modeled as a Rayleigh distribution. A pictorial view of LOS and NLOS scattering is depicted in Figure 2.21.

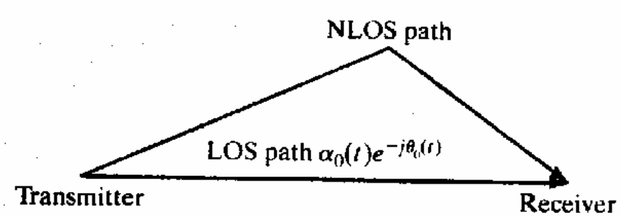

Figure 2.21 NLOS versus LOS scattering.

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Small-Scale Multipath Fading

- Given a channel with N scatterers, each with gain $\alpha_n(t)$ and delay $\tau_n(t)$
- Consider a digital transmission with carrier f_c and a symbol interval >> $\Delta\tau$ the delay spread

$$r(t) = \sum_{n=1}^{N} \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} x(t - \tau_n(t))$$

$$\approx \left[\sum_{n=1}^{N} \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} \right] x(t - \bar{\tau}).$$

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Small-Scale Multipath Fading

Let $Z(t) = Z_c(t) - jZ_s(t)$

$$Z_c(t) = \sum_{n=1}^{N} \alpha_n(t) \cos\theta_n(t)$$

$$Z_s(t) = \sum_{n=1}^{N} \alpha_n(t) \sin\theta_n(t),$$

where $\theta_n(t) = 2\pi f_c \tau_n(t)$.

Furthermore, let

$$\alpha(t) = \sqrt{Z_c^2(t) + Z_s^2(t)}, \quad \theta(t) = \tan^{-1}[Z_s(t)/Z_c(t)]$$

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2.5 Rayleigh Fading - NLOS

Assume that, at any time t, for $n = 1, 2, \ldots, N$,

a. the values of $\theta_n(t)$ are statistically independent, each being uniformly distributed over $[0,\ 2\pi]$;
b. the values of $\alpha_n(t)$ are identically distributed random variables, independent of each other and of the $\theta_n(t)$'s.

According to the central limit theorem, $Z_c(t)$ and $Z_s(t)$ are approximately Gaussian random variables at any time t if N is sufficiently large. For simplicity of notation, let Z_c and Z_s denote $Z_c(t)$ and $Z_s(t)$ at any time t. It can be shown that Z_c and Z_s are independent Gaussian random variables with zero mean and equal variance $\sigma_z^2 = \frac{1}{2}\sum_{n=1}^{N} E[\alpha_n^2]$, where α_n denotes $\alpha_n(t)$ at any time t. As a result, the joint pdf of Z_c and Z_s is

$$f_{Z_cZ_s}(x, y) = \frac{1}{2\pi\sigma_z^2} \exp\left[-\frac{x^2 + y^2}{2\sigma_z^2}\right], \quad -\infty < x < \infty, \ -\infty < y < \infty. \qquad (2.5.2)$$

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Rayleigh Fading - NLOS

a. the amplitude fading, α, follows a Rayleigh distribution with parameter σ_z^2,

$$f_\alpha(x) = \begin{cases} \dfrac{x}{\sigma_z^2} \exp\left(-\dfrac{x^2}{2\sigma_z^2}\right), & x \geq 0 \\ 0, & x < 0 \end{cases},$$

with $E[\alpha] = \sigma_z\sqrt{\pi/2}$ and $E(\alpha^2) = 2\sigma_z^2$;

b. the phase distortion follows a uniform distribution over $[0,\ 2\pi]$,

$$f_\theta(x) = \begin{cases} \dfrac{1}{2\pi}, & 0 \leq x \leq 2\pi \\ 0, & \text{elsewhere} \end{cases};$$

c. the amplitude fading α and the phase distortion θ are independent.

The channel is called a Rayleigh fading channel.

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Rician –LOS Propagation

Rician Fading (LOS propagation). If there exists an LOS path, the channel gain can be represented by

$$Z(t) = Z_c(t) - jZ_s(t) + \Gamma(t),$$

where $\Gamma(t) = \alpha_0(t)e^{-j\theta_0(t)}$ is the deterministic LOS component, and $Z_c(t) - jZ_s(t)$ represents all the NLOS components. With the LOS component, $E[Z(t)] = \Gamma(t) \neq 0$. The distribution of the envelope at any time t is given by the Rayleigh distribution modified by

a. a factor containing a non-centrality parameter, and
b. a zero-order modified Bessel function of the first kind.

The resultant pdf for the amplitude fading at any t, α, is known as the Rician distribution, given by (see Appendix D)

$$f_\alpha(x) = \underbrace{\frac{x}{\sigma_z^2}\exp\left(-\frac{x^2}{2\sigma_z^2}\right)}_{\text{Rayleigh}} \cdot \underbrace{\exp\left\{-\frac{\alpha_0^2}{2\sigma_z^2}\right\} \cdot I_0\left(\frac{\alpha_0 x}{\sigma_z^2}\right)}_{\text{modifier}}$$

$$= \frac{x}{\sigma_z^2}\exp\left(-\frac{x^2+\alpha_0^2}{2\sigma_z^2}\right) I_0\left(\frac{\alpha_0 x}{\sigma_z^2}\right), \quad x \geq 0, \qquad (2.5.5)$$

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Rician –LOS Propagation

where α_0 is $\alpha_0(t)$ at any t. α_0^2, is the power of the LOS component and is the non-centrality parameter, $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind and is given by

$$I_0(x) = \frac{1}{2\pi}\int_0^{2\pi} \exp(x\cos\theta)d\theta. \qquad (2.5.6)$$

The Rician fading channel has an important parameter called the K factor. It is defined as

$$K \triangleq \frac{\text{Power of the LOS component}}{\text{Total power of all other scattered components}} = \frac{\alpha_0^2}{2\sigma_z^2}.$$

As K approaches zero, the Rician distribution approaches the Rayleigh distribution. On the other hand, as K approaches infinity, only the dominant component matters and there is no fading. As a result, the wireless channel approaches an AWGN channel. Figure 2.22 shows the Rician distribution with $\sigma_z = 1$ and various K values. Assuming $\theta_0(t) = \pi/2$, it can be derived that, at a given t, the pdf of the carrier phase distortion $\theta(t)$ is given by

$$f_\theta(x) = \frac{1}{2\pi}\exp(-K) + \frac{1}{2}\sqrt{\frac{K}{\pi}}(\cos x)\exp(-K\sin^2 x)[1 + \text{erf}(\sqrt{K}\cos x)] \qquad (2.5.7)$$

for $x \in [-\pi, +\pi]$, where $\text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2}dy$ is the error function.

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Small-Scale Multipath Fading

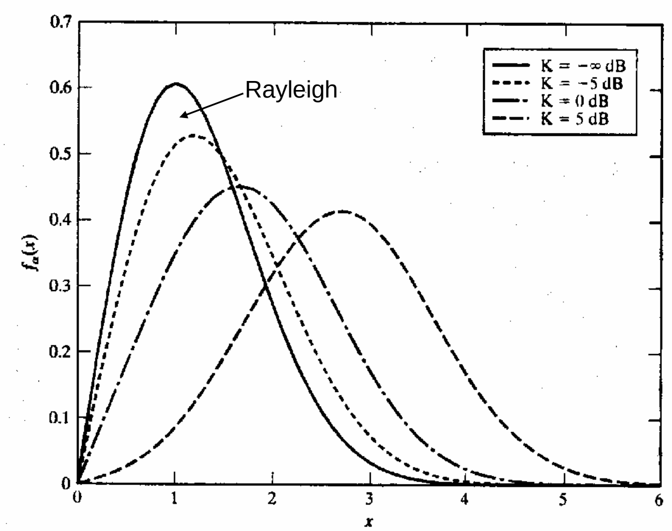

Figure 2.22 Rayleigh and Rician fading distributions with $\sigma_z = 1$.

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Propagation –
LCR and AFD

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Other Statistics

- The pdfs of the amplitude distortion $f_{\alpha}(x)$ and the phase distortion $f_{\theta}(x)$ explain how the signal will behave at each instant in time. They do not tell us how they change with time.

- We need to know how fast the channel fading changes with time.

- **LCR-** The Level Crossing Rate

- **AFD-** The Average Fade Duration

- LCR and AFD describe the frequency of fading

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LCR- Level Crossing Rate

Level Crossing Rate

Definition 2.3 The crossing rate at level R of a flat fading channel is the expected number of times that the channel amplitude fading level, $\alpha(t)$, crosses the specified level R, with a positive slope, divided by the observation time interval.

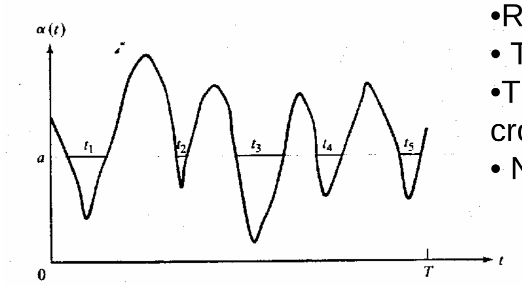

Figure 2.23 Level crossing rate and average duration of fade

- R is the chosen threshold
- The observation time is [0,T]
- The number of positive crossings is $M_T = 5$
- $N_R = M_T / T$ = # per second

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LCR

$$N_R = \text{E[upward crossing rate at level } R]. \quad (2.5.8)$$

Let $\dot{\alpha}$ denote the amplitude fading rate, $d\alpha(t)/dt$, at any time t, and let $f_{\alpha\dot{\alpha}}(x, y)$ denote the joint pdf of the amplitude fading $\alpha(t)$ and its derivative $\dot{\alpha}(t)$ at any time t. Then $f_{\alpha\dot{\alpha}}(x, y)|_{x=R}$ gives the joint pdf at the amplitude level R. From the definition, LCR is the expectation of the positive rate (i.e., $\dot{\alpha} > 0$) and at the level R, which can be expressed by

$$N_R = \int_0^\infty y f_{\alpha\dot{\alpha}}(x, y)|_{x=R} dy. \quad (2.5.9)$$

For the Rayleigh fading environment studied in Subsection 2.5.1, it can be shown that [130]

$$f_{\alpha\dot{\alpha}}(x, y) = \frac{x}{\sqrt{2\pi\sigma_{\dot{\alpha}}^2\sigma_z^2}} \exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_z^2} + \frac{y^2}{\sigma_{\dot{\alpha}}^2}\right)\right], \quad x \geq 0, -\infty < y < \infty, \quad (2.5.10)$$

where

$$\sigma_{\dot{\alpha}}^2 = \frac{1}{2}(2\pi \nu_m)^2\sigma_z^2$$

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LCR

ν_m is the maximum Doppler shift. Substituting Eq. (2.5.10) into Eq. (2.5.9), the LCR is

$$N_R = \int_0^\infty y \cdot \frac{R}{\sqrt{2\pi\sigma_{\dot{\alpha}}^2\sigma_z^2}} \exp\left[-\frac{1}{2}\left(\frac{R^2}{\sigma_z^2} + \frac{y^2}{\sigma_{\dot{\alpha}}^2}\right)\right] dy$$

$$= \sqrt{2\pi}\nu_m \left(\frac{R}{\sqrt{2}\sigma_z}\right) \exp\left(-\frac{R^2}{2\sigma_z^2}\right).$$

Letting

$$\rho = \frac{R}{\sqrt{2}\sigma_z}$$

be the normalized threshold with respect to the rms value of α (i.e., $\sqrt{2}\sigma_z$), we have

$$N_R = \sqrt{2\pi}\nu_m \rho \exp(-\rho^2).$$

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LCR

$$N_R = \sqrt{2\pi}\, \nu_m \rho \exp(-\rho^2)$$

The LCR is a product of two terms. The first term, $\sqrt{2\pi}\nu_m$, is proportional to the maximum Doppler shift. Since $\nu_m = \frac{V f_c}{c}$, where V is the velocity of the mobile user, f_c is the carrier frequency, and c is the speed of light, LCR is proportional to the user speed and the carrier frequency.

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LCR

$$N_R = \sqrt{2\pi}\, \nu_m \rho \exp(-\rho^2)$$

The second term, $\rho \exp(-\rho^2)$, depends only on the normalized threshold. Figure 2.24 shows how the component changes with the normalized threshold ρ in dB. It is observed that the maximum value for LCR occurs at a value of ρ which is 3 dB below the rms value.

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LCR

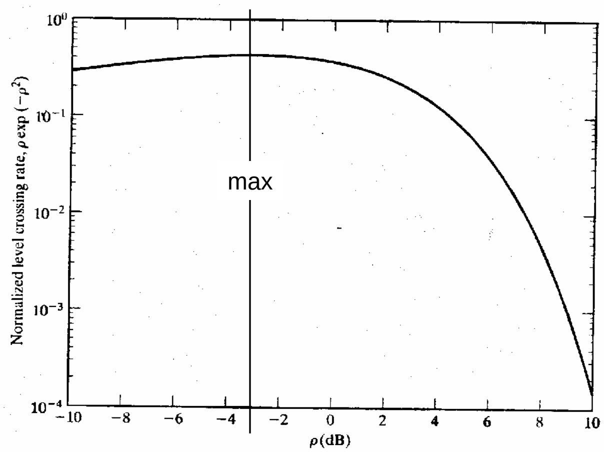

Figure 2.24 The normalized level crossing rate of the flat Rayleigh fading channel.

The maximum LCR is at ρ = -3 dB because the pdf of α is maximized at threshold

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AFD- Average Fade Duration

Definition 2.4 The average fade duration at level R is the average period of time for which the channel amplitude fading level is below the specified threshold R during each fade period.

Let χ_R denote the AFD. It is a statistic closely related to the LCR. Mathematically, the AFD can be represented as

$$\chi_R = \text{E[the period that the amplitude fading level stays below the threshold } R \text{ in each upward crossing].}$$

For the example shown in Figure 2.23, the AFD is $\sum_{i=1}^{5} t_i/5$. From the definitions of LCR and AFD, we have

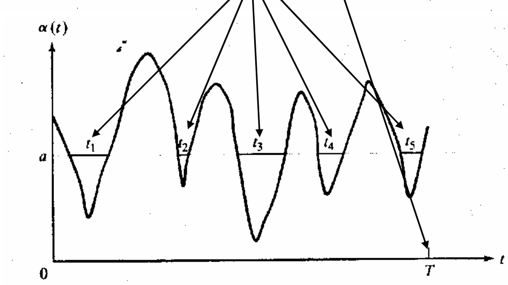

Figure 2.23 Level crossing rate and average duration of fade

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AFD- Average Fade Duration

$$N_R \cdot \chi_R = \lim_{T\to\infty} \frac{M_T}{T} \cdot \frac{\sum_{i=1}^{M_T} t_i}{M_T}$$

$$= \lim_{T\to\infty} \frac{\sum_{i=1}^{M_T} t_i}{T}$$

$$= P(\alpha \le R). \tag{2.5.12}$$

Equation (2.5.12) provides a relationship among the three statistics (i.e., LCR, AFD, and the cumulative distribution function (cdf) of the amplitude fading α). Thus, if any two statistics are known, the third one can also be determined. For the Rayleigh fading environment, the cdf of α is

$$P(\alpha \le x) = \int_0^x f_\alpha(y)\,dy = 1 - \exp\left(-\frac{x^2}{2\sigma_z^2}\right). \tag{2.5.13}$$

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AFD- Average Fade Duration

By Eqs. (2.5.11)–(2.5.13), the corresponding AFD is

$$\chi_R = \frac{P(A \le R)}{N_R}$$

$$= \frac{1 - \exp(-R^2/2\sigma_z^2)}{\sqrt{2\pi}\,\nu_m (R/\sqrt{2}\sigma_z)\exp(-R^2/2\sigma_z^2)}$$

$$= \frac{\exp(\rho^2) - 1}{\sqrt{2\pi}\,\nu_m \rho}. \tag{2.5.14}$$

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AFD- Average Fade Duration

$$\chi_R = \frac{\exp(\rho^2) - 1}{\sqrt{2\pi}\, \nu_m \rho}.$$

The AFD is a product of two components. The first component, $1/(\sqrt{2\pi}\,\nu_m)$, indicates that the AFD is inversely proportional to the mobile speed and the carrier frequency.

The second term, $[\exp(\rho^2) - 1]/\rho$, depends only on the normalized threshold ρ. Figure 2.26 shows how the component changes with the normalized threshold in dB. The value of the AFD increases dramatically as the threshold ρ increases much above the rms value. This can be explained from Figure 2.25. With a large threshold value, it is very unlikely for the amplitude level α to cross the threshold. Therefore, the length of time that α stays below the threshold can be very long.

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AFD- Average Fade Duration

Knowledge of the AFD value helps to determine the most likely number of bits that may be lost during a fade. This is useful for relating the received signal-to-noise ratio (SNR) during a fade to the instantaneous bit error rate (BER).

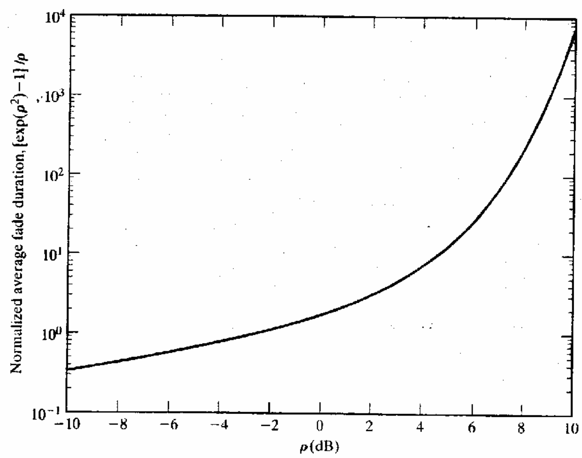

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Figure 2.26 The normalized average fade duration of the flat Rayleigh fading channel.