Futures Options

Chapter 16

Mechanics of Call Futures Options

When a call futures option is exercised the holder acquires

1. A long position in the futures
2. A cash amount equal to the excess of the futures price at previous settlement over the strike price

Mechanics of Put Futures Option

When a put futures option is exercised the holder acquires

1. A short position in the futures
2. A cash amount equal to the excess of the strike price over the futures price at previous settlement

The Payoffs

If the futures position is closed out immediately:

Payoff from call = $F - K$

Payoff from put = $K - F$

where F is futures price at time of exercise

Potential Advantages of Futures Options over Spot Options

- Futures contract may be easier to trade than underlying asset
- Exercise of the option does not lead to delivery of the underlying asset
- Futures options and futures usually trade in adjacent pits at exchange
- Futures options may entail lower transactions costs

Put-Call Parity for European Futures Options (Equation 16.1, page 347)

Consider the following two portfolios:

1. European call plus Ke^{-rT} of cash
2. European put plus long futures plus cash equal to F_0e^{-rT}

They must be worth the same at time T so that

$$c+Ke^{-rT}=p+F_0\,e^{-rT}$$

Other Relations

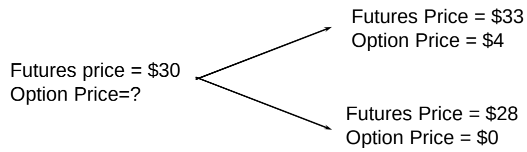
$$F_0 e^{-rT} - K < C - P < F_0 - Ke^{-rT}$$

$$c > (F_0 - K)e^{-rT}$$

$$p > (F_0 - K)e^{-rT}$$

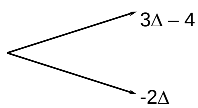
Binomial Tree Example

A 1-month call option on futures has a strike price of 29.



Setting Up a Riskless Portfolio

- Consider the Portfolio: long Δ futures
  short 1 call option



- Portfolio is riskless when $3\Delta - 4 = -2\Delta$ or $\Delta = 0.8$

Valuing the Portfolio
(Risk-Free Rate is 6%)

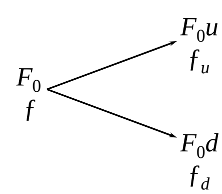
- The riskless portfolio is:
  long 0.8 futures
  short 1 call option
- The value of the portfolio in 1 month is
  –1.6
- The value of the portfolio today is
  $-1.6e^{-0.06/12} = -1.592$

Valuing the Option

- The portfolio that is
  long 0.8 futures
  short 1 option
  is worth –1.592
- The value of the futures is zero
- The value of the option must therefore be 1.592

Generalization of Binomial Tree Example (Figure 16.2, page 349)

- A derivative lasts for time T and is dependent on a futures



Generalization
(continued)

- Consider the portfolio that is long Δ futures and short 1 derivative

$$F_0 u\,\Delta - F_0\,\Delta - f_u$$

$$F_0 d\,\Delta - F_0\Delta - f_d$$

- The portfolio is riskless when

$$\Delta = \frac{f_u - f_d}{F_0 u - F_0 d}$$

Generalization
(continued)

- Value of the portfolio at time T is
  $F_0 u\,\Delta - F_0\Delta - f_u$
- Value of portfolio *today* is $-f$
- Hence
  $f = -[F_0 u\,\Delta - F_0\Delta - f_u]e^{-rT}$

Generalization
(continued)

- Substituting for Δ we obtain

$$f = [\,p\,f_u + (1-p)f_d\,]e^{-rT}$$

where

$$p = \frac{1-d}{u-d}$$

Growth Rates For Futures Prices

- A futures contract requires no initial investment
- In a risk-neutral world the expected return should be zero
- The expected growth rate of the futures price is therefore zero
- The futures price can therefore be treated like a stock paying a dividend yield of r
- This is consistent with the results we have presented so far (put-call parity, bounds, binomial trees)

Valuing European Futures Options

- We can use the formula for an option on a stock paying a continuous yield
  - Set S_0 = current futures price (F_0)
  - Set q = domestic risk-free rate (r)
- Setting $q = r$ ensures that the expected growth of F in a risk-neutral world is zero

Black's Model
(Equations 16.7 and 16.8, page 351)

- The formulas for European options on futures are known as Black's model

$$c = e^{-rT}\left[F_0\,N(d_1) - K\,N(d_2)\right]$$

$$p = e^{-rT}\left[K\,N(-d_2) - F_0\,N(-d_1)\right]$$

where $d_1 = \dfrac{\ln(F_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}}$

$$d_2 = \frac{\ln(F_0/K) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

How Black's Model is Used in Practice

- European futures options and spot options are equivalent when future contract matures at the same time as the otion.
- This enables Black's model to be used to value a European option on the spot price of an asset

Using Black's Model Instead of Black-Scholes (Example 16.5, page 352)

- Consider a 6-month European call option on spot gold
- 6-month futures price is 620, 6-month risk-free rate is 5%, strike price is 600, and volatility of futures price is 20%
- Value of option is given by Black's model with F_0=620, K=600, r=0.05, T=0.5, and σ=0.2
- It is 44.19

American Futures Option Prices vs American Spot Option Prices

- If futures prices are higher than spot prices (normal market), an American call on futures is worth more than a similar American call on spot. An American put on futures is worth less than a similar American put on spot
- When futures prices are lower than spot prices (inverted market) the reverse is true

Futures Style Options (page 353-54)

- A futures-style option is a futures contract on the option payoff
- Some exchanges trade these in preference to regular futures options
- The futures price for a call futures-style option is

$$F_0 N(d_1) - KN(d_2)$$

- The futures price for a put futures-style option is

$$KN(-d_2) - F_0 N(-d_1)$$

Put-Call Parity Results: Summary

Nondividend Paying Stock :

$$c + K e^{-rT} = p + S_0$$

Indices :

$$c + K e^{-rT} = p + S_0 e^{-qT}$$

Foreign exchange :

$$c + K e^{-rT} = p + S_0 e^{-r_f T}$$

Futures :

$$c + K e^{-rT} = p + F_0 e^{-rT}$$

Summary of Key Results from Chapters 15 and 16

- We can treat stock indices, currencies, & futures like a stock paying a continuous dividend yield of q
  - For stock indices, q = average dividend yield on the index over the option life
  - For currencies, $q = r_f$
  - For futures, $q = r$