EE334 - Magnetic Forces and Torques:

30 Lecture: pp. 205-213 5-1

Magnetostatics:

Electrostatic equations: $\nabla \cdot \vec{D} = \rho_V$, $\nabla \times \vec{E} = 0$ analogous to magnetostatic: $\nabla \cdot \vec{B} = 0$, $\nabla \times \vec{H} = \vec{J}$

Charge density (ρ_V) creates E-field Current Density (J) creates H-field

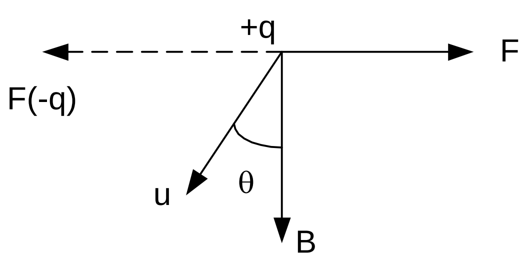
Magnetic flux density relates the permeability and magnetic field intensity

$$\vec{B} = \mu\ \vec{H}$$

Experimental data showed that a moving charge in a magnetic field created a magnetic force on the moving charge given by the relationship:

$\vec{F}_m = q\vec{u} \times \vec{B} \quad (N)$ B is the magnetic flux density (tesla T) T = N/(C-m/s)

The magnitude is then: $\left|\vec{F}_m\right| = quB\sin\theta$ where θ is the angle between B and u



this is for a positive charge if the charge is negative it is in the opposite direction

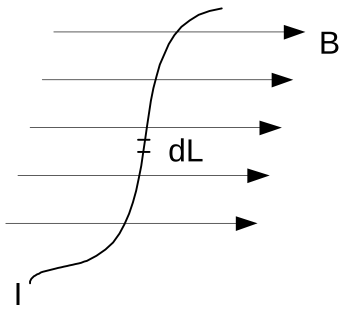
The total force on a charged particle is then the vector sum of the electrical component and the magnetic component.

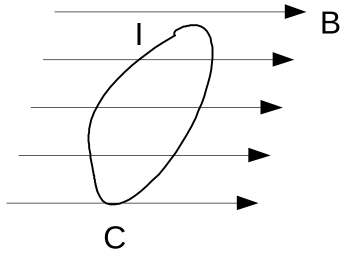
$\vec{F}_T = \vec{F}_E + \vec{F}_M = q\vec{E} + q\vec{u} \times \vec{B} = q\left(\vec{E} + \vec{u} \times \vec{B}\right)$ This is the Lorentz Force Equation

1. The electric force is in the direction of the electric field, and the magnetic force is perpendicular to the magnetic field and velocity
2. The magnetic force acts only on moving charges (currents), while the electric field acts on any charge whether moving or not.
3. Magnetic field does no work, no change in KE only direction no acceleration. $dW = \vec{F}_m \cdot d\vec{L} = \left(\vec{F}_m \cdot \vec{u}\right)dt = 0$

Can write moving charges in terms of current along a path (in a wire)

$$d\vec{F}_m = Id\vec{L} \times \vec{B}$$





if we have a constant current loop, the total force on the loop then becomes:

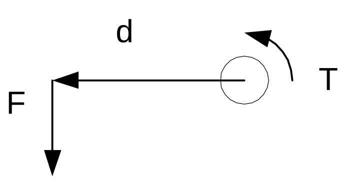
$$\vec{F}_m = I\oint_C d\vec{L} \times \vec{B}$$

if the magnetic field is uniform then it can come out of the integral and

$\vec{F}_m = I\left(\oint_C d\vec{L}\right) \times \vec{B} = 0$ The net force on a closed loop in a constant magnetic field is zero

so how do magnetic motors work if no net force? Create torque

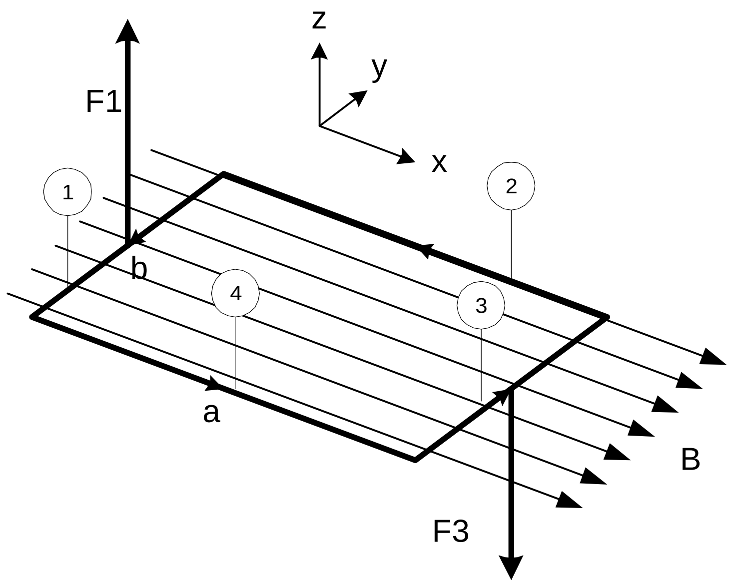
What is torque? Force on a moment arm



Can write this in vector notation: $\vec{\mathrm{T}} = \vec{d} \times \vec{F}$

$\vec{\mathrm{T}} = dF \sin\theta \ \hat{z}$ governed by right hand rule, thumb points in direction of torque when fingers point in direction of revolution.

Lets look at a current loop in a magnetic field:

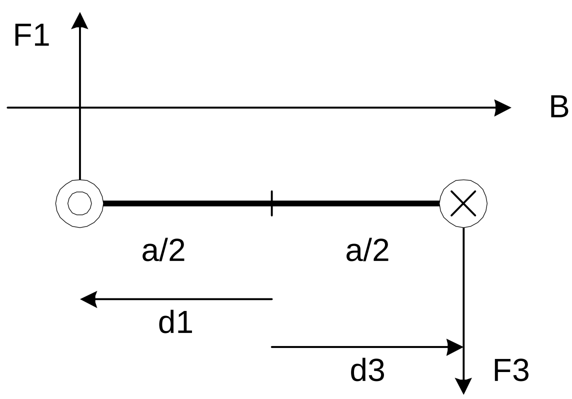


There is no force on leg 2 and leg 4, since the current is parallel to the magnetic field

The forces on the other two legs are:

$$\vec{F}_1 = I(-b\hat{y})\times(B\hat{x}) = IbB\hat{z}$$
$$\vec{F}_3 = I(b\hat{y})\times(B\hat{x}) = -IbB\hat{z}$$

Look at a side view:



The total torque then is:

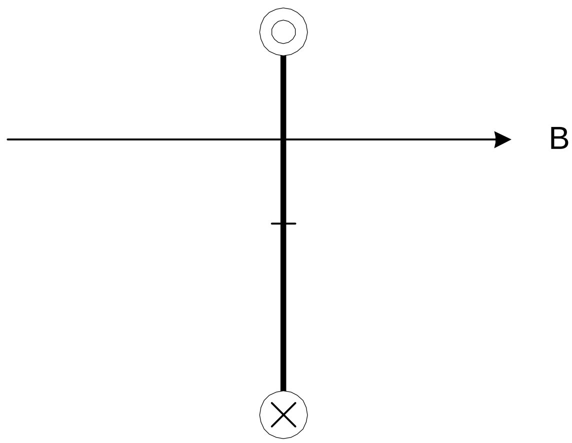
$$\vec{\mathrm{T}}_T = \vec{\mathrm{T}}_1 + \vec{\mathrm{T}}_3 = \vec{d}_1 \times \vec{F}_1 + \vec{d}_3 \times \vec{F}_3$$

$$= \left(-\frac{a}{2}\hat{x}\right) \times \left(IbB\hat{z}\right) + \left(\frac{a}{2}\hat{x}\right) \times \left(-IbB\hat{z}\right) = IabB\hat{y}$$

$$= IAB\hat{y}$$

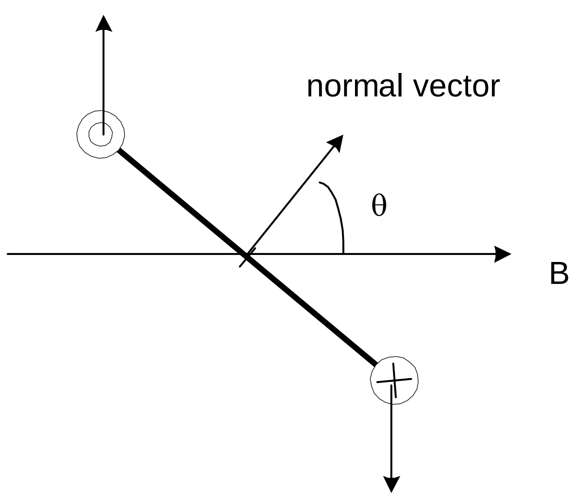
where A is the area of the loop.

What is the torque if the loop is rotated to this? Zero



The torque is zero when the B-Field is perpendicular to the plane of the loop.

How does the torque vary as a function of angle?



$$\left|\vec{\mathrm{T}}\right| = IAB\sin\theta$$

This is for a single loop, if we had a coil with N loops then the torque on the loop would be:

$$\mathrm{T} = NIAB\sin\theta$$

if we define the magnetic moment of a current carrying wire as

$$m \equiv NIA$$

and the direction normal to the plane of the wire,

$$\vec{m} = NIA\hat{n}$$

then the torque looks like a cross product between the magnetic moment and the B-field.

$$\vec{\mathrm{T}} = \vec{m} \times \vec{B}$$